Five Year Integrated M.Sc. Examination, 2024
Semester-VII
Subject: Integral Equations and Calculus of Variations
Course Code: MT-4-7-5

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt any four questions.

1. (a) Show that if λ_m and λ_n ($\lambda_m \neq \lambda_n$) are eigen values with correspondng eigen functions $y_m(x)$ and $y_n(x)$ then

$$\int_a^b y_m(x)y_n(x)dx = 0$$

(b) State and prove Hilbert-Schmidt theorem. 3+(2+5)=10

2. (a) Find Resolvent Kernel of

$$y(x) = f(x) + \lambda \int_0^x y(t)dt$$

hence solve it.
(b) Find eigen values and eigen functions of

$$y(x) = \lambda \int_0^{2\pi} \sin(x+t)y(t)dt$$

5+5=10

3. (a) State and prove Leibniz's formula.

(b) Convert the following integral equations into differential equation and associated conditions

$$y(t) = 5\cos(t) + \int_0^t (t-u)y(u)du$$

5+5=10

4. Solve the following integral equations:
(a)

$$y(x) = x^2 + \lambda \int_0^1 (1 - 3x\xi) y(\xi) d\xi,$$

(b)

$$u(x) = \sin x + \lambda \int_0^\pi \sin x \sin(2y) u(y) dy, ,$$

for all values of λ. 5+5=10

5. (a) Explain invalid variational problem. Test for an extremum of the functional

$$I(x) = \int_0^1 [2(x(t))^3 + 3t^2 x'(t)] dt$$

where $x(0) = 0$ and $x(1) = 1$.
(b) Show that the functional $\int_0^\pi (y'^2 - y^2) dx$, $y(0) = 0$, $y(\pi) = 0$ has infinite number of extremals.

6+4=10

6. (a) Find the shortest smooth plane curve joining two distinct points in the plane.

(b) By considering the Euler-Lagrange equation for $f(x, y, y')$ show that

$$\frac{d}{dx}\left\{ f - y' \frac{\partial f}{\partial y'} \right\} - \frac{\partial f}{\partial x} = 0$$

. 6+4=10